Name:

Spring 2018 Math 245 Exam 1

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 12:40 and will end at 1:30; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. $\binom{a}{b}$

b. floor

c. Commutativity theorem (for propositions)

d. Distributivity theorem (for propositions)

Problem 2. Carefully define the following terms:

a. Addition semantic theorem

b. Disjunctive Syllogism semantic theorem

c. contrapositive (proposition)

d. predicate

Problem 3. Prove or disprove: For all $n \in \mathbb{N}$, $(n-1)!|(n+1)!$.

Problem 4. Prove or disprove: For all odd a, b, $\frac{a+b}{2}$ is even.

Problem 5. Let p, q be propositions. Prove or disprove: $(p \downarrow q) \rightarrow (p \uparrow q)$ is a tautology.

Problem 6. Without using truth tables, prove the Destructive Dilemma theorem, which states: Let p, q, r, s be arbitrary propositions. Then $p \rightarrow q, r \rightarrow s, (\neg q) \vee (\neg s) \vdash (\neg p) \vee (\neg r)$.

Problem 7. Let $x \in \mathbb{R}$. Prove that if $2x \notin \mathbb{Q}$, then $3x + 1 \notin \mathbb{Q}$.

Problem 8. Let p, q, r, s be propositions. Simplify $\neg(((p \to q) \to r) \land s)$ as much as possible (where no compound propositions are negated).

Problem 9. Fix our domain to be $\mathbb{R}$. Simplify $\neg(\exists y\ \forall x\ \forall z\ (x < y) \to (x < z))$ as much as possible (where nothing is negated).

Problem 10. Prove or disprove: $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R},\ |y| \leq |y - x|$.